



General Certificate of Education  
Advanced Level Examination  
January 2010

# Mathematics

# MFP3

## Unit Further Pure 3

Tuesday 19 January 2010 9.00 am to 10.30 am

**For this paper you must have:**

- a 12-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 The function  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = x \ln(2x + y)$$

and

$$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with  $h = 0.1$ , to obtain an approximation to  $y(3.1)$ , giving your answer to four decimal places. (3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where  $k_1 = h f(x_r, y_r)$  and  $k_2 = h f(x_r + h, y_r + k_1)$  and  $h = 0.1$ , to obtain an approximation to  $y(3.1)$ , giving your answer to four decimal places. (5 marks)

2 (a) Given that  $y = \ln(4 + 3x)$ , find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . (3 marks)

(b) Hence, by using Maclaurin's theorem, find the first three terms in the expansion, in ascending powers of  $x$ , of  $\ln(4 + 3x)$ . (2 marks)

(c) Write down the first three terms in the expansion, in ascending powers of  $x$ , of  $\ln(4 - 3x)$ . (1 mark)

(d) Show that, for small values of  $x$ ,

$$\ln\left(\frac{4 + 3x}{4 - 3x}\right) \approx \frac{3}{2}x \quad (2 \text{ marks})$$

- 3 (a) A differential equation is given by

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 3x$$

Show that the substitution

$$u = \frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx} + \frac{2}{x}u = 3 \quad (2 \text{ marks})$$

- (b) Find the general solution of

$$\frac{du}{dx} + \frac{2}{x}u = 3$$

giving your answer in the form  $u = f(x)$ . (5 marks)

- (c) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 3x$$

giving your answer in the form  $y = g(x)$ . (2 marks)

- 4 (a) Write down the expansion of  $\sin 3x$  in ascending powers of  $x$  up to and including the term in  $x^3$ . (1 mark)

- (b) Find

$$\lim_{x \rightarrow 0} \left[ \frac{3x \cos 2x - \sin 3x}{5x^3} \right] \quad (4 \text{ marks})$$

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5 It is given that  $y$  satisfies the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^{-2x}$$

- (a) Find the value of the constant  $p$  for which  $y = pxe^{-2x}$  is a particular integral of the given differential equation. *(4 marks)*
- (b) Solve the differential equation, expressing  $y$  in terms of  $x$ , given that  $y = 2$  and  $\frac{dy}{dx} = 0$  when  $x = 0$ . *(8 marks)*

6 (a) Explain why  $\int_1^{\infty} \frac{\ln x^2}{x^3} dx$  is an improper integral. *(1 mark)*

(b) (i) Show that the substitution  $y = \frac{1}{x}$  transforms  $\int \frac{\ln x^2}{x^3} dx$  into  $\int 2y \ln y dy$ . *(2 marks)*

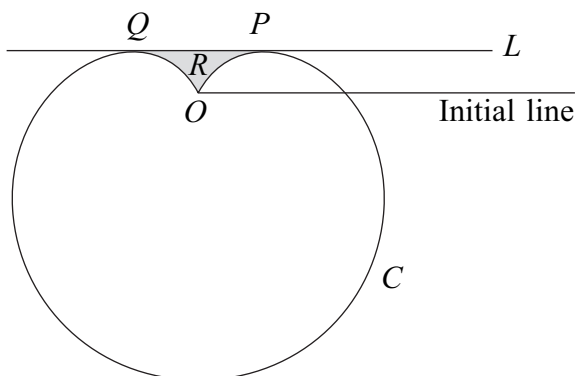
(ii) Evaluate  $\int_0^1 2y \ln y dy$ , showing the limiting process used. *(5 marks)*

(iii) Hence write down the value of  $\int_1^{\infty} \frac{\ln x^2}{x^3} dx$ . *(1 mark)*

7 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 8x^2 + 9 \sin x$$
 *(8 marks)*

- 8 The diagram shows a sketch of a curve  $C$  and a line  $L$ , which is parallel to the initial line and touches the curve at the points  $P$  and  $Q$ .



The polar equation of the curve  $C$  is

$$r = 4(1 - \sin \theta), \quad 0 \leq \theta < 2\pi$$

and the polar equation of the line  $L$  is

$$r \sin \theta = 1$$

- (a) Show that the polar coordinates of  $P$  are  $\left(2, \frac{\pi}{6}\right)$  and find the polar coordinates of  $Q$ .  
(5 marks)
- (b) Find the area of the shaded region  $R$  bounded by the line  $L$  and the curve  $C$ . Give your answer in the form  $m\sqrt{3} + n\pi$ , where  $m$  and  $n$  are integers.  
(11 marks)

**END OF QUESTIONS**

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